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Dissipativity-Based Synchronization of Mode-Dependent for Stochastic Complex Dynamical Networks with Semi- Markov Jump Topology, [1]

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Abstract: This paper considers the synchronization control for mode-dependent for stochastic Complex dynamical networks (CDNs) based on dissipativity theory. Particularly, the the network topologies of the CDNs are governed by the semi-markov process such that it may switch from one to another at different instants. By applying the Lyapunov-Krasovskii functional, Ito's formula, Jensen's single integral inequality, and some linear matrix inequalities, mode-dependent sufficient synchronization criterion are established while satisfying the desired dissipativity performance. Also, the control gain matrix can be obtained. An illustrate example is finally given to show the effectiveness advantages of the proposed theoretical results.
Key Words: Stochastic, Complex dynamical networks, Dissipativity, Semi-Markov jump topology, Lyapunov-Krasovskii functional, Synchronization, Linear matrix inequality.
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1. Introduction

At some point of the past two decades, the investigation of complex dynamical networks (CDNs) that consist of a huge number of interacting dynamical nodes, in which a node is a essential unit, that may have different meanings in different situations, consisting of chemical substrates, microprocessors, computer systems, schools, corporations, papers, webs, humans, and so on [1], [2]. Synchronization of complex networks of dynamical structures has received a great deal of attention from the nonlinear dynamics network. There are two types of connection between nodes: directed connection and undirected connection, the connection relationship can be unweighted and weighted. Many real-world systems can be described by complex dynamical networks, such as internet, neural network, biological system, traffic network.

Synchronization is one of the basic motions in nature where many connected systems evolve in synchrony and has been an ever hot research topic [3]. The synchronization phenomenon is a fundamental characteristic in nature [4]. The synchronization of complex dynamical networks have been significantly investigated in diverse fields of science and engineering due to its many potential practical applications [5]. Synchronization processes are ubiquitous in our lives, which play an critical role including synchronous communication, signal synchronization, firefly bioluminescence synchronization in biology, geostationary satellite, synchronous motor, database synchronization and so forth.

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Dissipativity was introduced in [6] and generalized in [7]. It is noteworthy that dissipativity concept can provide a powerful framework for the analysis and design of control systems using input–output description based on system energy related considerations. Dissipativity theory is an important idea which has been used in many areas of sciences and control engineering. Dissipative theory for dynamical structures became first initiated in [6], which has been generalized and considerably investigated for nonlinear systems in [7]. The dissipativity and passivity problems for a spread of sensible systems are attracting researchers' attention for several years [8]. Dissipativity theory introduces the system input and output descriptions and claims that it is a more general case of the H_∞ and the passivity performances [9].

On the other hand, the word "stochastic" means "pertaining to chance" (Greek roots), and is thus used to describe subjects that contain some element of random or stochastic behavior. stochastic systems have been successfully applied in modeling practical systems in many areas such as, biological, economical systems, and engineering, mainly due to the fact that stochastic disturbances exist universally in reality. In addition, noise disturbance is a major source of instability and can lead to poor performance in CDNs. Because in real nervous system, synaptic transmission is a noisy process brought on by random fluctuation from the release of neurotransmitters or by some random causes, so that stochastic perturbations are encountered at every level of human nervous systems. Moreover, [10] and [11] have shown that a neural network can be stabilized or destabilized by certain stochastic inputs. Because the great many applications of stochastic Ito's structures in real world [12], the study on feedback controller design for this kind of magnificence systems has received a great deal of attention; see [13] and the references therein.

It must be pointed out that the interconnection topology a number of the nodes of CDNs plays a significant role in the study of synchronization problem. Inside the existing literature, there had been two reported kinds of interconnection topologies, which might be constant/fixed topology [14] and time-varying topology [15]. Furthermore, as a special kind of hybrid systems [16], Markov jump systems (MJSs) [17] have shown some advantages of describing various physical systems, such as economics systems, manufacturing systems, power systems, network Recently, there has been some initial concern with the semi-Markov topology. based control systems, and other aspects. For positive systems with Markovian jump parameters [18], the problems of stochastic stability and mode-dependent state-feedback controller design in both continuous-time and discrete-time contexts were addressed. For the semi-Markov jump process, the sojourn time obeys a nonexponential distribution [19]. In other words, according to the transition probability distribution of the sojourn time, one may additionally conveniently that the Markov jump process is a unique case of the semi-Markov process to a point [20]. As a result, it is reasonable to investigate the CDNs subject to semi-Markov jump topology (SMJT) [21].

In this paper, we focus on the synchronization model of CDNs with semi-Markov jump topology stochastic noises and mode-dependent nodes, as well as the state feedback control design. More precisely, the topology is modulated by a continuous-time discrete-state semi-Markov process chain. The main contributions of this paper are mainly threefold: 1) The mode-dependent model with semi-Markov jump topology and external disturbances is proposed to describe more realistic dynamics of CDNs in practical applications. 2) More precisely, a new delay-dependent sufficient conditions are proposed in terms of LMIs by utilizing the Lyapunov stability theory and the stochastic analysis techniques. Subsequently, based on the developed condition, a design algorithm of the proposed state feedback controller to guarantee the stochastic $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ dissipativity performance is adopted for dealing with the corresponding synchronization problem. 3) Based on the idea that the transition rate $\pi_{\alpha\beta}(h)$ are transformed into polytopic-type uncertainties, the proposed synchronization criteria are expressed as low-dimensional LMIs, which could be checked readily and have not heavy computational resource demanding. 4) Eventually, a numerical example is shown to illustrate the effectiveness of the proposed theoretical results.

Notations: The notations throughout this paper are standard. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote n dimensional Euclidean space and the se of all $m \times n$ matrices, respectively $\mathcal{L}_2[0,\infty)$, $(\Omega, \mathcal{F}, \mathcal{P})$ is a complete probability space, Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation of the stochastic process or vector. $A - B > 0 (A - B < 0)$ denotes that $A - B$ is positive definite (negative definite). $A \otimes B$ stands for the Kronecker product. $*$ in a in a matrix denotes the elements below the main diagonal of a symmetric matrices, and $diag\{\cdots\}$ denotes a block-diagonal matrix.

2. Model Description and Preliminaries

Given a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, $\{\sigma(t), t \geq 0\}$ is a continuous-time discrete-state semi-Markov process taking values in a finite set $\mathcal{F} = \{1, 2, ..., \mathcal{N}\}$. The transition probability matrix $\Theta = (\pi_{\alpha\beta}(h), h > 0, \forall\alpha, \beta \in \mathcal{F}$ is defined by

$$Pr(\sigma(t+h) = \beta | \sigma(t) = \alpha) = \begin{cases} \pi_{\alpha\beta}(h)h + o(h), & if \quad \alpha \neq \beta \\ 1 + \pi_{\alpha\alpha}(h)h + o(h), & if \quad \alpha = \beta, \end{cases} \tag{1}$$

where $h > 0$ is the sojourn time, $lim_{h\to 0}(o(h)/h) = 0$ and $\pi_{\alpha\beta}(h) \geq 0$ for $\alpha \neq \beta$ is the transition rate from mode α at time t to mode β at time $t + h$ and

$$\pi_{\alpha\alpha}(h) = -\sum_{\beta=1,\beta\neq\alpha}^{\mathcal{N}} \pi_{\alpha\beta}(h), \quad \forall\alpha \in \mathcal{F}. \tag{2}$$

For notational simplicity, we hereafter denote the semi-Markov process parameter $\sigma(t)$ by α.

In this paper, we consider a class of directed complex dynamical networks (CDNs) with semi-Markov coupling and stochastic noise, which consists of N identical nodes and is defined over the Wiener process probability space $(\Omega, \mathcal{F}, \mathcal{P})$, such a network model can be described in the following form:

$$\begin{aligned} dx_z(t) = \Big[&\mathcal{A}(\sigma(t))x_z(t) + \mathcal{B}(\sigma(t))x_z(t-\tau(t)) + \mathcal{C}(\sigma(t))f(x_z(t)) + \mathcal{B}_1(\sigma(t))f(x_z(t-\tau(t))) \\ &+ \sum_{j=1}^{N} \mathcal{G}_{zj}(\sigma(t))\Gamma(\sigma(t))x_j(t) + u_z(t) + \mathcal{D}(\sigma(t))v_z(t)\Big]dt + \rho(t, x_z(t), x_z(t-\tau(t)))d\omega(t), \quad z = 1, 2, ...N, \end{aligned} \tag{3}$$

where $x_z(t) = (x_{z1}(t), x_{z2}(t), ..., x_{zn}(t))^T \in \mathbb{R}^n$ denotes the sate vector of the zth node; $u_z \in \mathbb{R}^n$ and $v_z \in \mathbb{R}^q$ denotes the control input and the disturbance input on the zth node, respectively; $f(x_z(t)) \in \mathbb{R}^n$ is a smooth nonlinear function; $\tau(t)$ is the time-varying delay; $\mathcal{A}(\sigma(t))$ is a diagonal matrix; $\mathcal{B}(\sigma(t))$, $\mathcal{C}(\sigma(t))$, $\mathcal{B}_1(\sigma(t))$ and $\mathcal{D}(\sigma(t))$ are weight matrices; $\Gamma(\sigma(t))$ is the inner coupling matrix; and $\mathcal{G}(\sigma(t)) = (\mathcal{G}_{zj}(\sigma(t))) \in \mathbb{R}^{N\times N}$ is the outer coupling matrix representing the directed network topology. If there is a directed coupling from node z to node $j(z \neq j)$, then the coupling $\mathcal{G}_{zj}(\sigma(t)) \neq 0$; otherwise, $\mathcal{G}_{zj}(\sigma(t)) = 0$. Moreover, $\mathcal{G}_{zz}(\sigma(t))$ is defined by $\mathcal{G}_{zz}(\sigma(t)) = -\sum_{j=1,j\neq z}^{N} \mathcal{G}_{zj}(\sigma(t))$, $z = 1, 2, ..., N$. The function $\rho(\cdot,\cdot,\cdot) : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the noise intensity function matrix; $\omega(t)$ is a 1-dimensional Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with $\mathbb{E}\{\omega(t)\} = 0$, $\mathbb{E}\{\omega^2(t)\} = 1$ and $\mathbb{E}\{w(s)w(t)\} = 0$ for $s \neq t$, where $\mathbb{E}$ is the mathematical expectation. Without loss of generality, the initial conditions are $x_z(t) = \Phi_z(t)$, $t \in [-\overline{\tau}, 0]$.

Assumption 1. For each $\sigma(t) \in \mathcal{F}$, the network topology keeps constant, and $\mathcal{A}(\sigma(t))$, $\mathcal{B}(\sigma(t))$, $\mathcal{C}(\sigma(t))$, $\mathcal{B}_1(\sigma(t))$ $\mathcal{D}(\sigma(t))$ and $\Gamma(\sigma(t))$ are known real constant matrices.

Assumption 2. For all $x, y \in \mathbb{R}^n$, the continuous nonlinear functions f satisfy the following sector-bounded conditions,

$$[f(x) - f(y) - F_1(x - y)]^T [f(x) - f(y) - F_2(x - y)] \leq 0,$$

where F_1, F_2 are real constant matrices with $F_2 - F_1 \geq 0$.

Assumption 3. The time varying delay $\tau(t)$ satisfies

$$0 < \tau(t) \leq \overline{\tau},$$

where $\overline{\tau}$ is a positive constant.

Assumption 4. The noise intensity function $\rho(t, e(t), e(t - \tau(t)))$ satisfies the Lipschitz condition and there exists a appropriate dimensions positive constant matrices ϕ_1 and ϕ_2 such that

$$trace\Big[\rho(t, e(t), e(t - \tau(t)))\Big]^T \times \Big[\rho(t, e(t), e(t - \tau(t)))\Big] \leq \vartheta_1 e^T(t)\phi_1 e(t) + \vartheta_2 e^T(t - \tau(t))\phi_2 e(t - \tau(t)),$$

where ϑ_1 and ϑ_2 are known nonnegative constants.

Remark 1. It can be found that the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative synchronization problem is a more general case of $\mathcal{H}_\infty$ and passivity synchronization by adjusting $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ matrices.

In this paper, the synchronization errors are defined as

$$e_z(t) = x_z(t) - s(t), \quad z = 1, 2, ...N, \tag{4}$$

where $s(t) \in \mathbb{R}^n$ is the state trajectory of the unforced isolated node

$$ds(t) = \Big[\mathcal{A}(\sigma(t))s(t) + \mathcal{B}(\sigma(t))s(t-\tau(t)) + \mathcal{C}(\sigma(t))f(s(t)) + \mathcal{B}_1(\sigma(t))f(s(t-\tau(t)))\Big]dt. \tag{5}$$

Then, the synchronization error dynamics of the CDNs can be obtained as follows:

$$\begin{aligned} de_z(t) = \Big[&\mathcal{A}(\sigma(t))e_z(t) + \mathcal{B}(\sigma(t))e_z(t-\tau(t)) + \mathcal{C}(\sigma(t))\widetilde{f}(e_z(t)) + \mathcal{B}_1(\sigma(t))\widetilde{f}(e_z(t-\tau(t))) \\ &+ \sum_{j=1}^{N}\mathcal{G}_{zj}(\sigma(t))\Gamma(\sigma(t))e_j(t) + u_z(t) + \mathcal{D}(\sigma(t))v_z(t)\Big]dt + \widehat{\rho}(t, e_z(t), e_z(t-\tau(t)))d\omega(t), \quad z = 1, 2, ...N, \end{aligned} \tag{6}$$

where $\widetilde{f}(e_z(t)) = f(x_z(t)) - f(s(t))$, $\widetilde{f}(e_z(t-\tau(t))) = f(x_z(t-\tau(t))) - f(s(t-\tau(t)))$.

The following mode-dependent synchronization controller s designed:

$$u_z(t) = \mathcal{K}(\sigma(t))e_z(t), \tag{7}$$

where $\mathcal{K}(\sigma(t))$ is the controller gain matrix.

Consequently, the closed-loop form of the error system can be obtained as follows:

$$\begin{aligned} de_z(t) = \Big[&(\mathcal{A}(\sigma(t)) + \mathcal{K}(\sigma(t)))e_z(t) + \mathcal{B}(\sigma(t))e_z(t-\tau(t)) + \mathcal{C}(\sigma(t))\widetilde{f}(e_z(t)) + \mathcal{B}_1(\sigma(t))\widetilde{f}(e_z(t-\tau(t))) \\ &+ \sum_{j=1}^{N}\mathcal{G}_{zj}(\sigma(t))\Gamma(\sigma(t))e_j(t) + \mathcal{D}(\sigma(t))v_z(t)\Big]dt + \widehat{\rho}(t, x_z(t), e_z(t-\tau(t)))d\omega(t), \quad z = 1, 2, ..., N, \end{aligned} \tag{8}$$

By using the Kronecker product properties and mathematical manipulations, the error system (8) can be written in the following compact form:

$$\begin{aligned} de(t) = \Big[&[I_N \otimes \mathcal{A}(\sigma(t)) + \mathcal{K}(\sigma(t))]e_z(t) + [I_N \otimes \mathcal{B}(\sigma(t))]e_z(t-\tau(t)) + [I_N \otimes \mathcal{C}(\sigma(t))]\widetilde{f}(e_z(t)) + [I_N \otimes \mathcal{B}_1(\sigma(t))] \\ &\widetilde{f}(e_z(t-\tau(t))) + [\mathcal{G}(\sigma(t)) \otimes \Gamma(\sigma(t))]e(t) + [I_N \otimes \mathcal{D}(\sigma(t))]v(t)\Big]dt + \widehat{\rho}(t, e(t), e(t-\tau(t)))d\omega(t), \end{aligned} \tag{9}$$

where

$$\begin{aligned} e(t) &= \Big[e_1^T(t), e_2^T(t), ..., e_N^T(t)\Big]^T, \\ F(e(t)) &= \Big[\widetilde{f}^T(e_1(t)), \widetilde{f}^T(e_2(t)), ..., \widetilde{f}^T(e_N(t))\Big]^T, \\ F(e(t-\tau(t))) &= \Big[\widetilde{f}^T(e_1(t-\tau(t))), \widetilde{f}^T(e_2(t-\tau(t))), ..., \widetilde{f}^T(e_N(t-\tau(t)))\Big]^T, \\ v(t) &= \Big[v_1^T(t), v_2^T(t), ..., v_N^T(t)\Big]^T, \\ \widehat{\rho}(t, e(t), e(t-\tau(t))) &= \Big[\widehat{\rho}(t, e_1(t), e_1(t-\tau(t))), \widehat{\rho}(t, e_2(t), e_2(t-\tau(t))), ..., \widehat{\rho}(t, e_N(t), e_N(t-\tau(t)))\Big]^T. \end{aligned} \tag{10}$$

For simplicity of notations we denote $\mathcal{A}(\sigma(t))$, $\mathcal{B}(\sigma(t))$, $\mathcal{C}(\sigma(t))$, $\mathcal{B}_1(\sigma(t))$ $\mathcal{D}(\sigma(t))$, $\mathcal{G}(\sigma(t))$, $\Gamma(\sigma(t))$, by $\mathcal{A}_\alpha$, $\mathcal{B}_\alpha$, $\mathcal{C}_\alpha$, $\mathcal{B}_{1\alpha}$, $\mathcal{D}_\alpha$, $\mathcal{G}_\alpha$, Γ_α, for $\sigma(t) = \alpha \in \mathcal{F}$. Then one has

$$de(t) = \Big[(I_N \otimes (\mathcal{A}_\alpha \otimes \mathcal{K}_\alpha))e(t) + (I_N \otimes \mathcal{B}_\alpha)e(t-\tau(t)) + (I_N \otimes \mathcal{C}_\alpha)F(e(t)) + (I_N \otimes \mathcal{B}_{1\alpha})F(e(t-\tau(t))) + (\mathcal{G}_\alpha \otimes \Gamma_\alpha)e(t) + (I_N \otimes \mathcal{D}_\alpha)v(t)\Big] + \widehat{\rho}(t, e(t), e(t-\tau(t)))d\omega(t). \tag{11}$$

The following definition is given.

Definition 2.1. *Given real symmetric matrices $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ with appropriate dimensions, the system (11) is called strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})-\gamma$-dissipative if there exists a scalar $\gamma > 0$ such that for any $t_p > 0$, following condition holds with zero initial condition:*

$$\mathbb{E}\Big\{\langle e(t), \mathcal{Q}e(t)\rangle_{t_p} + 2\langle e(t), \mathcal{S}v(t)\rangle_t + \langle v(t), \mathcal{R}v(t)\rangle_{t_p}\Big\} \geq \mathbb{E}\Big\{\gamma\langle v(t), v(t)\rangle_{t_p}\Big\}, \tag{12}$$

where $\langle e(t), \mathcal{Q}e(t)\rangle_{t_p}$, denotes $\int_0^t e^T(s)\mathcal{Q}e(s)ds$, and the other symbols are similarly defined.

Before proceeding further, the following lemma is introduced for subsequent analysis.

Lemma 2.2. *[22] For any constant matrix $M \in \mathbb{R}^{n\times n}$, $M^T = M > 0$, scalars α and β with $\alpha > \beta$ and vector $x : [\beta, \alpha] \to \mathbb{R}^n$, such that the following integrations are well defined, then*

$$-(\alpha-\beta)\int_\beta^\alpha x^T(s)Mx(s)ds \leq \Big(\int_\beta^\alpha x(s)ds\Big)^T M\Big(\int_\beta^\alpha x(s)ds\Big),$$

$$\frac{(\alpha-\beta)^2}{2}\int_\beta^\alpha\int_u^\alpha x^T(s)Mx(s)dsdu \leq -\Big(\int_\beta^\alpha\int_u^\alpha x(s)dsdu\Big)^T M\Big(\int_\beta^\alpha\int_u^\alpha x(s)dsdu\Big)$$

3. Main Results

In this section, we derive a new mode-dependent $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ dissipative criterion for synchronization controller with stochastic CDNs (1) in the following theorem. For presentation convenience, in the following, we denote

Theorem 3.1. *Assume that Assumptions [1]-[4] hold. For given scalars $\overline{\tau}$ and γ, the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ dissipative in the sense of Definition 2.1, synchronization of the stochastic CDNs (3) can be achieved with given mode-dependent synchronization controller gain $\mathcal{K}_\alpha$, if there exist mode-dependent matrix $\mathcal{P}_\alpha > 0$ and $\mathcal{V}_\alpha$, matrices $\mathcal{W}_1 > 0$, $\mathcal{W}_2 > 0$ and positive scalars $\vartheta > 0$, $\beta_1 > 0, \beta_2 > 0$ such that $\Pi_\alpha < 0$ for each $\alpha \in \mathcal{F}$, where*

$$\mathcal{P}_\alpha < \vartheta I, \tag{13}$$

$$\Pi_\alpha = \Big[\Pi\Big]_{7\times 7} < 0, \tag{14}$$

where

$$\Pi_{1,1} = 2(I_N \otimes \mathcal{P}_\alpha\mathcal{A}_\alpha + I_N \otimes \mathcal{V}_\alpha) + 2(\mathcal{G}_\alpha \otimes \mathcal{P}_\alpha\Gamma_\alpha) + \vartheta(I_N \otimes \mathcal{R}_3) + \sum_{\beta=1}^N \pi_{\alpha\beta}(h)(I_N \otimes \mathcal{P}_\beta) + (I_N \otimes \mathcal{W}_1) + \overline{\tau}^2(I_N \otimes \mathcal{W}_2) - \beta_1(F_1^TF_2 + F_2^TF_1) + \mathcal{Q}^T\mathcal{Q},\ \Pi_{1,2} = (I_N \otimes \mathcal{P}_\alpha\mathcal{B}_\alpha),$$

$$\Pi_{1,4} = (I_N \otimes \mathcal{P}_\alpha \mathcal{C}_\alpha) + \beta_1(F_1^T + F_2^T),\ \Pi_{1,5} = (I_N \otimes \mathcal{P}_\alpha \mathcal{D}_\alpha) - \mathcal{S},\ \Pi_{1,7} = (I_N \otimes \mathcal{B}_{1\alpha}),$$

$$\Pi_{2,2} = \vartheta(I_N \otimes \mathcal{R}_4) - \beta_2(F_1^T F_2 + F_2^T F_1),\ \Pi_{2,7} = \beta_2(F_1^T + F_2^T),\ \Pi_{3,3} = -(I_N \otimes \mathcal{W}_1),$$

$$\Pi_{4,4} = -2\beta_1 I,\ \Pi_{5,5} = -\mathcal{R} + \gamma I,\ \Pi_{6,6} = -(I_N \otimes \mathcal{W}_2),\ \Pi_{7,7} = -2\beta_2 I.$$

Furthermore, the mode-dependent synchronization controller gain $\mathcal{K}_\alpha$ can be obtained by $\mathcal{K}_\alpha = \mathcal{P}_\alpha^{-1}\mathcal{V}_\alpha$.

Proof: *For each α, to develop the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity stochastic synchronization CDN model (3). It is enough to establish the stochastic stability criterion for the closed loop error system (11). For this purpose, we select the Lyapunov-Krasovskii functional as follows:*

$$V(e_t, \alpha, t) = \sum_{l=1}^{3} V_l(e_t, \alpha, t), \tag{15}$$

where

$$V_1(e_t, \alpha, t) = e^T(t)(I_N \otimes \mathcal{P}_\alpha)e(t), \tag{16}$$

$$V_2(e_t, \alpha, t) = \int_{t-\overline{\tau}}^{t} e^T(\varphi)(I_N \otimes \mathcal{W}_1)e(\varphi)d\varphi, \tag{17}$$

$$V_1(e_t, \alpha, t) = \overline{\tau} \int_{\overline{\tau}}^{0} \int_{t+\eta}^{t} e^T(\varphi)(I_N \otimes \mathcal{W}_2)e(\varphi)d\varphi d\eta. \tag{18}$$

Then, it follows from Ito's formula that

$$dV(e_t, t, \alpha) = \mathcal{L}V(e_t, t, \alpha) + V_e(e_t, t, \alpha)\rho(t, e(t), e(t-\tau(t)))d\omega(t), \tag{19}$$

where

$$\mathcal{L}V(e_t, t, \alpha) = \mathcal{L}V_1(e_t, t, \alpha) + \mathcal{L}V_2(e_t, t, \alpha) + \mathcal{L}V_3(e_t, t, \alpha) \text{ and } V_e(e_t, t, \alpha) = \frac{\partial V(e_t, t, \alpha)}{\partial e}.$$

Now, by calculating the time derivative of $V(e_t, t, \alpha)$ along the solution trajectories of the error system (11), we can get

$$\begin{aligned}
\mathcal{L}V_1(e_t, t, \alpha) = {} & 2e^T(t)(I_N \otimes \mathcal{P}_\alpha)\Big[(I_N \otimes (\mathcal{A}_\alpha + \mathcal{K}_\alpha))e(t) + (I_N \otimes \mathcal{B}_\alpha)e(t-\tau(t)) + (I_N \otimes \mathcal{C}_\alpha)F(e(t)) \\
& + (I_N \otimes \mathcal{B}_{1\alpha})F(e(t-\tau(t))) + (\mathcal{G}_\alpha \otimes \Gamma_\alpha)e(t) + (I_N \otimes \mathcal{D}_\alpha)v(t)\Big] + e^T(t)\sum_{\beta=1}^{N} \pi_{\alpha\beta}(h)(I_N \otimes \mathcal{P}_\beta)e(t) \\
& + trace\Big\{\rho^T(t, e(t), e(t-\tau(t)))(I_N \otimes \mathcal{P}_\alpha)\rho(t, e(t), e(t-\tau(t)))\Big\}, \qquad (20) \\
= {} & 2e^T(t)(I_N \otimes \mathcal{P}_\alpha \mathcal{A}_\alpha + I_N \otimes \mathcal{P}_\alpha \mathcal{K}_\alpha)e(t) + 2e^T(t)(I_N \otimes \mathcal{P}_\alpha \mathcal{B}_\alpha)e(t-\tau(t)) \\
& + 2e^T(t)(I_N \otimes \mathcal{P}_\alpha \mathcal{C}_\alpha)F(e(t)) + 2e^T(t)(I_N \otimes \mathcal{P}_\alpha \mathcal{B}_{1\alpha})F(e(t-\tau(t))) + 2e^T(t)(\mathcal{G}_\alpha \otimes \mathcal{P}_\alpha \Gamma_\alpha)e(t) \\
& + 2e^T(t)(I_N \otimes \mathcal{P}_\alpha \mathcal{D}_\alpha)v(t) + e^T(t)\sum_{\beta=1}^{N} \pi_{\alpha\beta}(h)(I_N \otimes \mathcal{P}_\beta)e(t)
\end{aligned}$$

$$+\, trace\Big\{\rho^T(t,e(t),e(t-\tau(t)))(I_N\otimes\mathcal{P}_\alpha)\rho(t,e(t),e(t-\tau(t)))\Big\}, \tag{21}$$

$$\mathcal{L}V_2(e_t,t,\alpha)=e^T(t)(I_N\otimes\mathcal{W}_1)e(t)-e^T(t-\overline{\tau})(I_N\otimes\mathcal{W}_1)e(t-\overline{\tau}), \tag{22}$$

$$\mathcal{L}V_3(e_t,t,\alpha)=\overline{\tau}^2e^T(t)(I_N\otimes\mathcal{W}_2)e(t)-\overline{\tau}\int_{t-\overline{\tau}}^{t}e^T(\varphi)(I_N\otimes\mathcal{W}_2)e(\varphi)d\varphi. \tag{23}$$

By Lemma 2.2, holds that

$$-\overline{\tau}\int_{t-\overline{\tau}}^{t}e^T(\varphi)(I_N\otimes\mathcal{W}_2)e(\varphi)d\varphi\le-\int_{t-\overline{\tau}}^{t}e^T(\varphi)d\varphi(I_N\otimes\mathcal{W}_2)\int_{t-\overline{\tau}}^{t}e(\varphi)d\varphi. \tag{24}$$

From Assumption 4, and condition (13), we have

$$\begin{aligned}trace\Big\{\rho^T(t,e(t),e(t-\tau(t)))(I_N\otimes\mathcal{P}_\alpha).\rho(t,e(t),e(t-\tau(t)))\Big\}&\le\vartheta\Big[trace\Big\{\rho^T(t,e(t),e(t-\tau(t)))\\&\quad\rho(t,e(t),e(t-\tau(t)))\Big\}\Big],\\&\le\vartheta\Big[e^T(t)(I_N\otimes\mathcal{R}_3)e(t)+e(t-\tau(t))\\&\quad(I_N\otimes\mathcal{R}_4)e(t-\tau(t))\Big],\end{aligned} \tag{25}$$

where ϑ are positive scalars $\mathcal{R}_3,\mathcal{R}_4$ are known constant matrices.

According to Assumption 2, for $\forall\ \beta_1>0,\beta_2>0$, it can be obtained that

$$\beta_1\begin{bmatrix}e(t)\\F(e(t))\end{bmatrix}^T\begin{bmatrix}F_1^TF_2+F_2^TF_1 & -F_1^T-F_2^T\\-F_1-F_2 & 2I\end{bmatrix}\begin{bmatrix}e(t)\\F(e(t))\end{bmatrix}\le0, \tag{26}$$

and

$$\beta_2\begin{bmatrix}e(t-\tau(t))\\F(e(t-\tau(t)))\end{bmatrix}^T\begin{bmatrix}F_1^TF_2+F_2^TF_1 & -F_1^T-F_2^T\\-F_1-F_2 & 2I\end{bmatrix}\begin{bmatrix}e(t-\tau(t))\\F(e(t-\tau(t)))\end{bmatrix}\le0. \tag{27}$$

Using (21) − (25) in (19) and adding (26) and (27) to (19), we have

$$dV(e_t,t,\alpha)-e^T(t)\mathcal{Q}e(t)-2e^T(t)\mathcal{S}v(t)-v^T(t)(\mathcal{R}-\gamma I)v(t)\le\eta^T(t)\Big(\Pi_{7\times7}\Big)\eta(t), \tag{28}$$

where

$$\eta^T(t)=\Big[e^T(t)\ e^T(t-\tau(t))\ e^T(t-\overline{\tau})\ F^T(e(t))\ v^T(t)\int_{t-\overline{\tau}}^{t}e^T(\varphi)d\varphi\ F^T(e(t-\tau(t)))\Big].$$

Based on Definition 2.1, if $\Pi_\alpha<0$. Then by taking the mathematical expectation to (28), we have,

$$\mathbb{E}\Big\{dV(e_t,t,\alpha)-e^T(t)\mathcal{Q}e(t)-2e^T(t)\mathcal{S}v(t)-v^T(t)(\mathcal{R}-\gamma I)v(t)\le\eta^T(t)\Big\}\le0. \tag{29}$$

Because $V(e_0, 0, \alpha) = 0$, under zero initial condition, that is $e_0 = 0$, for $t \in [\overline{\tau}, 0]$ by integrating (29) over the time period 0 to t_p, we have

$$2\int_0^{t_p} \mathbb{E}\Big\{e^T(s)\mathcal{S}v(s)\Big\}ds \geq \mathbb{E}\Big\{V(e_t, t, \alpha)\Big\} - \mathbb{E}\Big\{V(e_0, 0, \alpha)\Big\} - \mathcal{Q}\int_0^{t_p} \mathbb{E}\Big\{e^T(s)e(s)\Big\}ds$$

$$-(\mathcal{R} - \gamma I)\int_0^{t_p} \mathbb{E}\Big\{v^T(s)v(s)ds\Big\} \geq -\int_0^{t_p} \mathbb{E}\Big\{e^T(s)e(s)ds\Big\}. \tag{30}$$

Therefore by Definition 2.1 the considered stochastic CDNs (11) with time-varying delays is dissipative. This complete the proof.

4. Numerical Example

In this section, a numerical example is presented to demonstrate the effectiveness of the proposed synchronization stochastic control scheme for the complex dynamical network (3).

Example 4.1: Consider the CDNs (3) with three nodes ($N = 3$), where each node is two dimensional ($n = 2$). For the case of two modes ($\sigma(t) = 1, 2$), the parameters of the stochastic CDNs are given as

Mode 1.

$$\mathcal{A}_1 = \begin{bmatrix} -0.1 & 0.1 \\ 0 & -0.2 \end{bmatrix}, \ \mathcal{B}_1 = \begin{bmatrix} -0.1 & 0.1 \\ 0 & -0.5 \end{bmatrix}, \ \mathcal{C}_1 = \begin{bmatrix} -0.1 & 0.1 \\ 0 & -0.2 \end{bmatrix}, \ \mathcal{D}_1 = \begin{bmatrix} -2.5 & 0.3 \\ 0.9 & -1 \end{bmatrix},$$

$$\mathcal{B}_{11} = \begin{bmatrix} 0.3 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \ \Gamma_1 = \begin{bmatrix} 0.7 & -0.6 \\ -0.4 & 0.7 \end{bmatrix},$$

Mode 2.

$$\mathcal{A}_2 = \begin{bmatrix} -2.8 & 0.9 \\ 2.7 & -4.3 \end{bmatrix}, \ \mathcal{B}_2 = \begin{bmatrix} 0.5 & 0.9 \\ 2.7 & -4.3 \end{bmatrix}, \ \mathcal{C}_2 = \begin{bmatrix} 0.8 & -1.2 \\ -1 & 1.1 \end{bmatrix}, \ \mathcal{D}_2 = \begin{bmatrix} 0.1 & 1 \\ 0.5 & 1 \end{bmatrix},$$

$$\mathcal{B}_{12} = \begin{bmatrix} 0 & 0.3 \\ -0.1 & 0.2 \end{bmatrix}, \ \Gamma_2 = \begin{bmatrix} -0.3 & 0.25 \\ -0.35 & -0.4 \end{bmatrix}.$$

Where the outer coupling matrix $\mathcal{G}(\sigma(t)) = (\mathcal{G}_{ij}(\sigma(t)))$ can be accordingly obtained by

$$\mathcal{G}_1 = \begin{bmatrix} -3 & 1 & 2 \\ 2 & -3 & 1 \\ 1 & 2 & -3 \end{bmatrix}, \ \mathcal{G}_2 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

The noise intensity function vector σ is given by

$$\rho(t, e(t), e(t-\tau(t))) = \begin{bmatrix} -0.05 & 0.05 & 0.1 & 0.1 \\ 0.05 & -0.05 & 0.1 & -0.1 \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\tau(t)) \end{bmatrix}.$$

The time-varying delay is set by $\tau(t) = 0.15 + 0.05sint$, such that one has $\overline{\tau} = 0.2$, $\gamma = 0.1$. The nonlinear functions

$$f(x_i(t)) = \begin{bmatrix} 0.5x_{i1}(t) - tanh(0.2x_{i1}(t)) + 0.2x_{i2}(t) \\ 0.95x_{i2}(t) - tanh(0.75x_{i2}(t)) \end{bmatrix},$$

such that

$$F_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}, \ F_2 = \begin{bmatrix} 0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}.$$

In the simulation, the dissipative matrices are given as,

$$\mathcal{Q} = \begin{bmatrix} -0.9 & 0 \\ 0 & -0.9 \end{bmatrix}, \ \mathcal{S} = \begin{bmatrix} 0.5 & 0 \\ 0.3 & 1 \end{bmatrix}, \ \mathcal{R} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$

and the external disturbances are set as $v_i(t) = 0.5sin(10t)$. The transition rate chosen as $\pi_{\alpha\beta}(h) = \pi_{11}(h) = \pi = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$. With the above parameters, it can be verified that (31) has a feasible solutions as

$$P_1 = \begin{bmatrix} 0.0368 & 0.1273 \\ 0.1273 & 0.3070 \end{bmatrix}, \ \mathcal{P}_2 = \begin{bmatrix} 0.0389 & -0.0179 \\ -0.0179 & -0.2136 \end{bmatrix}, \ \mathcal{W}_1 = (10)^3 \begin{bmatrix} 4.2394 & -0.0007 \\ -0.0007 & 4.3246 \end{bmatrix},$$

$$\mathcal{W}_2 = (10)^3 \begin{bmatrix} 4.2394 & -0.0007 \\ -0.0007 & 4.3246 \end{bmatrix}, \ \beta_1 = (10)^3(1.6756), \ \beta_2 = 0.0017, \ \vartheta = (10)^4(5.8774),$$

and desired mode-dependent stochastic synchronization controller gains can be calculated as follows

$$\mathcal{K}_1 = (10)^5 \begin{bmatrix} 3.0439 & -1.2743 \\ -1.2569 & 0.3806 \end{bmatrix}, \ \mathcal{K}_2 = (10)^3 \begin{bmatrix} -1.2008 & 0.1057 \\ 0.1007 & 0.2304 \end{bmatrix}.$$

Therefore, by Theorem 3.1, the complex dynamical networks with time-varying delays achieve mode-dependent synchronization through the state feedback controller.

5. Conclusion

This paper deals with the dissipativity-based synchronization for mode-dependent CDNs with semi-Markov jump topology. Based on model transformation and stochastic noise, sufficient conditions are given for guaranteeing the synchronization with the prescribed dissipativity performance. In particular, we have considered the semi-Markov topology to obtain the synchronization criterion. Then the mode-dependent synchronization controllers are design. By employing the Lyapunov-Krasovskii stability theory and some stochastic analysis techniques. We then have developed a stochastic synchronization criterion for the considered network in terms of linear matrix inequalities and have presented a design algorithm for the proposed state feedback controller to a solution of the obtained set of linear matrix inequalities. Which can be checked numerically using the effective LMI toolbox in MATLAB. Finally, numerical examples have been presented to illustrate the advantages and applicability of the proposed results.

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